

## KINETICS OF MASS TRANSFER BETWEEN A POLYDISPERSE PARTICLE SYSTEM AND THE SURROUNDING MEDIUM

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There is a whole series of processes in which the principal role is played by the kinetics of heat and mass transfer between some polydisperse system of particles and the surrounding medium. These processes include, for example, the burning of fuel in pulverized or liquid-droplet form in a gaseous medium, heterogeneous chemical reactions, proceeding in different phases and accompanied by diffusion and heat flows to the particles, various processes of heat and mass transfer in fluidized beds, extraction processes, the evaporation of droplet or particulate aerosols, the dissolving of solid particles, processes of oil degasification, and so on. In describing the kinetics of such processes it is usual to employ results obtained from an examination of the boundary value problem for the equations of diffusion and heat conduction in the neighborhood of an individual particle. However, as a rule, the heat and mass transfer rates for a multi-particle medium depend, apart from the characteristics of each particle, on the state of the medium in which the particles are located. Since the state of the medium, in turn, depends on the kinetics of the processes at each particle, the problem of heat and mass transfer of the particle system becomes nonlinear, even if the corresponding problem for a single particle is linear. For example, the rate of burning of an aerosol depends on the composition of the gas, which varies with time [1, 2], the chemical reactions are determined by the concentration of reactants and reaction products in the medium, which also depend on the history of the process, and so on. All this leads to serious difficulties in computing various quantities of practical importance (e.g., the amount of fuel consumed or reaction product formed). The problem becomes even more complicated if the system of particles participating in a heterogeneous process is polydisperse, and interest attaches to certain more detailed characteristics of the system such as: the mean diameter and distribution function of the particles, the variation of this function with time, etc. Moreover, in [3] it was shown in relation to a particular example that it is impossible to interpret a real process involving a polydisperse system of particles with the help of certain relations for a monodisperse system, even in cases where only the integral characteristics of the process are of interest.

In this paper attention is concentrated primarily on the question of making an accurate allowance for the polydispersity, and secondly, on the elimination of the difficulties associated with the above-mentioned nonlinearity. Therefore we shall consider one of the simplest problems of this kind, namely, the evaporation of a system of droplets in a turbulent gaseous medium, when the investigation of the evaporation kinetics of a single droplet in an infinite gas can be reduced to the solution of a single equation of parabolic type in a space with an excluded sphere. This problem is also of considerable independent interest: among its possible applications it is sufficient to mention the physics of clouds and the evaporation of various artificial fogs, created in the chemical industry and in laboratory practice.

The problem of evaporation of a polydisperse system of droplets was previously solved in [3]. A certain generalization of the equation obtained in that paper is presented in the first section of this article. We note that the formulation of the problem adopted here is very similar to that of [3].

**§1. Formulation of the problem.** We shall assume that the concentration of the vapor and the temperature are roughly the same at all points in space. Strictly speaking, this assertion is valid only for droplets in a sufficiently turbulent medium; it is also usually assumed to hold in the case of a cloud, etc. Moreover,

for simplicity we shall assume that the temperature is such that the concentration of saturated vapor is much less than the density of the medium (this assumption is valid, as a rule, in real conditions). This enables us, firstly, to neglect local variations of temperature at the surface of the droplets, assuming that only the average temperature of the system as a whole varies, and secondly, to disregard Stefan flow and the resulting variation in the rate of evaporation of individual droplets. We also neglect the possible coagulation of droplets and assume that their characteristic dimensions are such that it is possible to disregard the dependence of the saturated vapor pressure on droplet radius. These assumptions impose definite limitations on the dimensions of the droplets of the system, which can be described with the aid of the relations obtained below.

Since, by assumption, the temperature is remote from boiling point, it is also possible to neglect the effects, considered in [4], of a decrease in droplet radius in the process of evaporation, and for the radius  $r$  of the individual droplet to write the Maxwellian expression

$$\frac{dr}{dt} = \frac{D}{r\rho} (q(t) - q^*(t)). \quad (1.1)$$

Here  $D$  is the coefficient of diffusion of vapor in the medium,  $\rho$  is the density of the liquid,  $q^*(t)$  is the density of the saturating vapor at a given temperature  $T(t)$ , and  $q(t)$  is the density of the vapor.

As in [3], we describe the polydispersity of the system with the help of the spectral function

$$\begin{aligned} dn(r) &= n_0 f(r, t) dr, \\ f(r, 0) &= f_0(r), \quad \int_0^\infty f_0(r) dr = 1. \end{aligned} \quad (1.2)$$

Here  $n_0$  is the number of droplets per unit volume at the initial moment. The change of  $f(r, t)$  with time formally corresponds to the flow of a certain fictitious liquid with density  $f(r, t)$ , for which the continuity equation holds. Thus, we get following analog of the kinetic equation in the region  $r > 0$ :

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial r} \left( f \frac{dr}{dt} \right) = \frac{\partial f}{\partial t} + \frac{D}{\rho} \frac{q(t) - q^*(t)}{r} \left( \frac{\partial f}{\partial r} - \frac{f}{r} \right) = 0. \quad (1.3)$$

The solution of this equation has the form

$$\begin{aligned} f(r, t) &= r (r^2 + \theta(t))^{-1/2} f_0 [(r^2 + \theta(t))^{1/2}], \\ \theta(t) &= \frac{2D}{\rho} \int_0^t [q^*(\tau) - q(\tau)] d\tau. \end{aligned} \quad (1.4)$$

The problem can be considered solved if it is possible to find the function  $\theta(t)$ . We shall use the method of [3]. From the condition of constant total content of evaporating substance per unit volume we have

$$q(t) = q_0 + \frac{4}{3} \pi \rho n_0 \left[ \langle r_0^3 \rangle - \int_0^\infty f_0 [(r^2 + \theta)^{3/2}] \frac{r^2 dr}{(r^2 + \theta)^{3/2}} \right]. \quad (1.5)$$

Here it is assumed that the volume occupied by the droplets is relatively small; here and henceforth the subscript zero relates to the initial moment, and the symbol  $\langle \rangle$  denotes averaging over the distribution  $f_0(r)$ . In accordance with the definition of  $\theta(t)$ , from (1.4) we have

$$q(t) = q^*(t) - \frac{\rho}{2D} \frac{d\theta}{dt}. \quad (1.6)$$

The quantity  $q^*(t)$  may be assumed to be a function of the mean temperature  $T(t)$ . However, from the heat balance equation it follows that

$$T(t) - T_0 \approx - \frac{L}{c} (q(t) - q_0).$$

Here  $L$  is the specific heat of evaporation,  $c$  is the specific heat per unit volume of the two-phase system at constant pressure. Thus,  $q^*(t)$  is a function of  $q(t)$ . Eliminating here and from (1.6) the function  $q^*(t)$ , we obtain a functional relation between  $q(t)$  and the derivative  $d\theta/dt$ . Substituting in (1.5), we obtain for  $\theta(t)$  the analog of the equation of [3] with allowance for the variation of temperature during evaporation. For the majority of real systems this equation can be simplified by assuming that the reduction in temperature due to evaporation is not large, so that  $q^*(t)$  can be represented in the form of the first few terms of a Taylor expansion with respect to the difference  $q(t) - q_0$ . Then  $q(t)$  is a linear function of  $d\theta/dt$ . We have

$$q(t) = \alpha q_0^* - \beta \frac{d\theta}{dt}, \quad \alpha = \frac{1 + bLq_0/cq_0^*}{1 + bL/c}, \quad (1.7)$$

$$\beta = \frac{\rho}{2D(1 + bL/c)}.$$

The quantity  $b$  is the coefficient in the expansion of  $q^*(T)$  with respect to the difference  $T(t) - T_0$  associated with the first power of this difference. Instead of (1.5), after substitution of the variable of integration it is easy to obtain

$$\frac{d\theta}{dt} = \frac{\alpha q_0^* - q_0}{\beta} + \frac{4}{3\beta} \pi \rho n_0 \left[ \int_{\sqrt{\theta}}^\infty (\rho^2 - \theta(t))^{3/2} f_0(\rho) d\rho - \langle r_0^3 \rangle \right]. \quad (1.8)$$

The solution of this equation is difficult even for the simplest forms of the initial spectral function  $f_0(r)$ . However, as in [3], for  $\theta$  it is easy to obtain approximate formulas corresponding to large and small initial unsaturations, asymptotic formulas for large  $t$ , etc.

An exception is the evaporation of a monodisperse system of droplets, when the solution of (1.8) is obtained without difficulty. In fact, in this case  $f_0(r) = \delta(r - r_0)$ , and after a series of transformations we get for  $\theta(t)$  the algebraic equation ( $\theta(t) < r_0$ )

$$t = 2 \int_{r_0}^{\sqrt{\theta}} \frac{\omega d\omega}{A + B\omega^2}, \quad r(\theta) = (r_0^2 - \theta(t))^{1/2},$$

$$A = \frac{\alpha q_0^* - q_0}{\beta} - \frac{4\pi\rho n_0 r_0^3}{3\beta}, \quad B = \frac{4\pi\rho n_0}{3\beta}.$$

If at a certain moment  $t_0$  the quantity  $\theta(t_0)$  is compared with  $r_0$  (this is possible only if  $A > 0$ ), then at this moment the system of droplets disappears, and at  $t > t_0$  we have

$$f(r, t) \equiv 0, \quad q(t) \equiv q_0 + \frac{4}{3} \pi \rho n_0 r_0^3,$$

$$T(t) \equiv T_0 - \frac{4}{3} \pi \rho n_0 r_0^3 L c^{-1}.$$

If  $A < 0$ , then, as may easily be seen, as  $t \rightarrow \infty$  equilibrium is attained between the droplets and the medium, the asymptotic radius of the droplets being easily computed from (1.8) with  $d\theta/dt = 0$ ; we have

$$r_\infty = r_0 - \{r_0^2 - [r_0^2 - (\frac{4}{3} \pi \rho n_0)^{-1} (\alpha q_0^* - q_0)]^{1/2}\}^{1/2}.$$

Hence and from the balance equations it is possible to express  $q_\infty$ ,  $T_\infty$  also. The results can be extended to the case when the system consists of several monodisperse fractions; however, in more general cases Eq. (1.8) gives practically nothing. Therefore it is of interest to investigate the kinetic equation (1.3) from other viewpoints.

**§2. Self-similar systems.** It is known that in a number of cases, when the fluctuations of the external conditions are small, the spectrum of the system of droplets at any moment of time is characterized by constant quantities—the mean radius, the volume of the droplets, etc. remain unchanged; only the number of droplets per unit volume  $n(t)$  changes, i.e., the system is self-similar. It is natural to seek a solution  $f(r, t)$  of Eq. (1.3) for such a system in the form of the product  $\Phi(t)R(r)$ . Substituting this in (1.3), we obtain

$$\frac{d\Phi}{dt} - \frac{D}{\rho} \Phi (q^*(t) - q(t)) \frac{1}{rR} \left( \frac{dR}{dr} - \frac{R}{r} \right) = 0. \quad (2.1)$$

Setting  $dR/dr - R/r = -2srR$ , where  $s$  is a constant, we have

$$R(r) = 2sr \exp(-sr^2). \quad (2.2)$$

The problem for  $\Phi(t)$  has the form

$$\frac{d\Phi}{dt} + \frac{2D}{\rho} s (q^*(t) - q(t)) \Phi = 0, \quad \Phi(0) = 1. \quad (2.3)$$

Condition (1.2), with (2.3), gives  $R(r) = f_0(r)$ , which determines the constant  $s$  in (2.2). From (1.6) and (1.7) we have

$$q^*(t) - q(t) = \frac{\rho}{2D\beta} (\alpha q_0^* - q(t)).$$

Hence and from (1.5) we get

$$2D\rho^{-1}s (q^*(t) - q(t)) = a + b\Phi(t), \quad \langle r_0^3 \rangle = \frac{3}{8} \sqrt{\pi s^{-3/2}},$$

$$a = \frac{s}{\beta} (\alpha q_0^* - q_0 - \frac{4}{3} \pi \rho n_0 \langle r_0^3 \rangle), \quad b = \frac{4s}{3\beta} \pi \rho n_0 \langle r_0^3 \rangle.$$

Thus, Eq. (2.3) is written in the form

$$d\Phi/dt + a\Phi + b\Phi^2 = 0.$$

This is the ordinary Bernoulli; its solution for  $\Phi(0) = 1$  is

$$\Phi(t) = [(1 + b/a)e^{at} - b/a]^{-1}. \quad (2.4)$$

We shall consider some special cases. 1. Let  $a > 0$  be the initial "water content" less than the initial unsaturation; at  $t \gg a^{-1}$  we have  $\Phi(t) \approx ae^{-at} (a + b)^{-1}$ . 2. At  $a = 0$  we get the special case when the system evaporates entirely, the equality  $q(t) = q^*(t)$  being satisfied as  $t \rightarrow \infty$ . The quantity  $\Phi(t)$  decreases to zero in accordance with the law  $\Phi(t) = (1 + bt)^{-1}$ . 3. If  $a < 0$ ,  $a + b > 0$  the initial "water content" is greater than the initial unsaturation; the system does not evaporate completely. The limiting value of  $\Phi(t)$  as  $t \rightarrow \infty$  is equal to  $|a|/b$ . If  $a + b = 0$ ,  $q(t) \equiv q^*(t) \equiv q_0$ , i.e., the system does not change at all. When  $a + b < 0$  we have  $q(t) > q^*(t)$ , which corresponds to condensation growth of the droplets in the supersaturated medium. However, since, when condensation is considered, Eq. (1.3), written in the region  $r > 0$ , becomes unsuitable (it must be supplemented on the right by a term allowing for the appearance of new droplets in the system), the results corresponding to the latter case have little physical significance.

All the other quantities characterizing the system may be expressed in terms of  $\Phi(t)$ :

$$\begin{aligned} n(t) &= n_0 \Phi(t), & q(t) &= q_0 + \gamma_0 (1 - \Phi(t)), \\ T(t) &= T_0 - L\gamma_0 c^{-1} (1 - \Phi(t)), & \gamma_0 &= 4/3 \pi \rho n_0 \langle r_0^3 \rangle. \end{aligned}$$

Here  $\gamma_0$  is the initial "water content." The specific heat  $c$  is expressed in the form of a sum

$$c = c_0 \gamma_0 + c_1 \gamma_1 + c_2 \gamma_2 \approx \text{const.}$$

The terms of the sum correspond to liquid vapor and gas;  $\gamma_i$  are the contents of these substances by weight per unit volume,  $c_i$  are their specific heats. A simple analysis shows that the maximum reduction of temperature occurs when  $\gamma_0 = \alpha q_0^*$ . In this case

$$\lim_{t \rightarrow \infty} (T_0 - T(t)) = \alpha L c^{-1} q_0^*$$

depends very strongly on the absolute initial temperature.

The relations obtained have comparatively simple form. From the purely practical viewpoint it is quite unnecessary, of course, for the true spectral function to belong to the one-parameter family (2.2). It is perfectly sufficient to extrapolate it from one of the functions of this family, with a certain error permitted by the conditions of the specific problem. If such extrapolation is possible, the relations described will characterize the behavior of the system with sufficient accuracy.

The spectral function of self-similar systems are a sort of eigenfunctions of the nonlinear problem considered. In view of the nonlinearity the direct application of the Fourier method to the solution of the problem for systems with functions  $f_0(r)$  not belonging to (2.2) is impossible. However, it is possible to generalize the method somewhat by reducing the problem to the solution of a certain system of ordinary differential equations. Let

$$f_0(r) = 2 \sum_{n=1}^N s_n c_n r \exp(-s_n r^2), \quad \sum_{n=1}^N c_n = 1. \quad (2.5)$$

Functions of type (2.5) can be used directly, for example, to describe various natural and artificial systems in whose spectrum for various reasons there is more than one maximum. We shall find  $f(r, t)$  in the form

$$f(r, t) = 2r \sum_{n=1}^N s_n c_n \Phi_n(t) \exp(-s_n r^2). \quad (2.6)$$

Substituting this expression in (1.3), we obtain, as before, the system

$$\frac{d\Phi_n}{dt} + \frac{2D}{\rho} s_n (q^*(t) - q(t)) \Phi_n = 0, \quad \Phi_n(0) = 1. \quad (2.7)$$

Hence, in particular, we immediately obtain the relations

$$s_m \ln \Phi_n \equiv s_n \ln \Phi_m, \quad \Phi_n^{s_m} \equiv \Phi_m^{s_n}.$$

Assuming that  $s_m > s_n$ , if  $m > n$ , we derive the important qualitative conclusion that with the passage of time the more distant maxima of the spectral function (corresponding to large  $r$ ) are smoothed out more rapidly than the nearer ones, i.e., the moments  $\langle r^k \rangle$  decrease with increase in  $t$  (and do not increase, notwithstanding the assertions of [3]). It is also clear that to solve the problem it is sufficient to find only one of the functions  $\Phi_n(t)$ —to be definite we shall find  $\Phi_1(t)$ , corresponding to the smallest  $s_n$  from (2.7),

$$\frac{d\Phi_1}{dt} = -\Phi_1 \left( a_1 + \sum_{n=1}^N b_1^{(n)} \Phi_1^{s_n/s_1} \right). \quad (2.8)$$

The constants  $a_1$  and  $b_1^{(n)}$  are easily computed in the same way as  $a$  and  $b$ ,

$$a_1 = \frac{s_1}{\beta} \left( \alpha q_0^* - q_0 - \pi^{3/2} \rho n_0 \sum_{n=1}^N \frac{c_n}{s_n^{3/2}} \right), \quad b_1^{(n)} = \frac{\pi^{3/2} \rho n_0 s_1}{\beta} \frac{c_n}{s_n^{3/2}}.$$

For arbitrary  $c_n$  and  $s_n$  this equation can be integrated only numerically. In the special case when  $s_n = k_n s_1$ , where  $k_n$  are integers, the right side of (2.8) is a polynomial, which can always be expanded in factors, and Eq. (2.8) integrated. The solution is then expressed in the form of a transcendental relation between  $\Phi_1$  and  $t$ . In this case, obviously, the root is taken equal to unity at  $t = 0$ . For example, in the case  $s_2 = 2s_1$ ,  $s_n = 0$  when  $n > 2$  we get

$$\begin{aligned} \ln \frac{\Phi_1^2}{\Phi_1^2 + \tau_1 \Phi_1 + \tau_2} - \tau_1 \int \frac{d\Phi_1}{\Phi_1^2 + \tau_1 \Phi_1 + \tau_2} &= -2a_1 t \\ \tau_1 &= \frac{2\sqrt{2}c_1}{1-c_1}, & \tau_2 &= \left[ \frac{(\alpha q_0^* - q_0)(2s)^{3/2}}{\pi^{3/2} \rho n_0 (1-c_1)} - 1 - \frac{2\sqrt{2}c_1}{1-c_1} \right]. \end{aligned}$$

Here it has been assumed that  $b_1^{(2)} \neq 0$ . Depending on the sign of the determinant

$$\Delta = 4\tau_2 - \tau_1^2$$

we get different laws of variation of  $\Phi(t)$ , describing the evaporation under different possible initial conditions.

**§3. Extension to systems with an arbitrary initial spectral function.** The results of the preceding section suggest the idea of finding  $f(r, t)$  for arbitrary  $f_0(r)$  in the form of an integral transform of some function  $\Phi(s, t)$  with a kernel belonging to the family (2.2). Let

$$f(r, t) = r \int_0^\infty A(s) e^{-sr^2} \Phi(s, t) ds. \quad (3.1)$$

Imposing on  $\Phi(s, t)$  condition  $\Phi(s, 0) = 1$ , we get, in particular,

$$f_0(r) = r \int_0^\infty A(s) e^{-sr} ds.$$

Or, introducing the new argument  $\sigma = r^2$ ,

$$\varphi(\sigma) = \int_0^\infty A(s) e^{-s\sigma} ds, \quad \varphi(\sigma) = \frac{f_0(r)}{r} \Big|_{r=\sqrt{\sigma}}. \quad (3.2)$$

The representation of (3.2) immediately enables us to answer the question for what  $\varphi(\sigma)$  it is possible. Thus, the only limitation imposed on the type of function  $\varphi(\sigma)$  consists in the requirement that  $\varphi(\sigma)$  be a Laplace transform of some other function  $A(s)$ , i. e., have an inverse transform. The condition of existence of an inverse transform is well known; therefore, henceforth, the question of the degree of generality will not be considered. Consequently, the function  $A(s)$  may be assumed known. By analogy with the previous argument, it is appropriate to represent the kernel in the form  $\Phi(s, t) = \Phi^S(t)$ , i. e., to write instead of (3.1)

$$f(r, t) = r \int_0^\infty A(s) e^{-sr} \Phi^S(t) ds. \quad (3.3)$$

After transformations for  $\Phi(t)$  we get the equation

$$\begin{aligned} \frac{d\Phi}{dt} &= -\Phi \left( a - \frac{4\pi\rho n_0}{3\beta} \int_0^\infty ds \int_0^\infty A(s) [1 - \Phi^S(t)] r^4 e^{-sr} dr \right) = \\ &= -\Phi \left( a_0 + \frac{4\pi\rho n_0}{3\beta} \int_0^\infty ds \int_0^\infty A(s) \Phi^S(t) r^4 e^{-sr} dr \right) \end{aligned} \quad (3.4)$$

Here

$$a_0 = \frac{\alpha q_0^* - q_0}{\beta} - \frac{4\pi\rho n_0}{3\beta} \langle r_0^3 \rangle = \frac{1}{\beta} (\alpha q_0^* - q_0 - \gamma_0).$$

Integrating with respect to  $r$  in (3.4), we finally obtain

$$\frac{d\Phi}{dt} = -\Phi \left( a_0 + \frac{\pi^{1/2} \rho n_0}{2\beta} \int_0^\infty s^{-1/2} \Phi^S(t) A(s) ds \right). \quad (3.5)$$

The chief advantage of Eq. (3.5) as compared with (1.8) consists in the fact that in the integral in (3.5) the limits of integration do not depend on the unknown function, and the integrand itself is, as a rule, simpler than in (1.8). Even if (3.5) is difficult to solve in finite form, its solution can always be obtained numerically. For real forms of  $f_0(r)$  the analogous process may prove inapplicable to (1.8).

As an example, while bearing in mind possible applications to the physics of clouds, we shall consider the case when the initial spectral function is given by the known Khrgian-Mazin formula [5]

$$f_0(r) = 2\omega^2 r e^{-r\omega}.$$

The corresponding function  $\varphi(\sigma)$  has the form

$$\varphi(\sigma) = 2\omega^2 \sqrt{\sigma} \exp(-\omega \sqrt{\sigma}). \quad (3.6)$$

Hence we have the following function:

$$A(s) = \frac{\omega^2 - 2s}{2\pi^{1/2} s^{3/2}} \omega^2 \exp\left(-\frac{\omega^2}{4s}\right). \quad (3.7)$$

Substituting (3.7) and (3.3) and integrating for  $\Phi(t) = 1$ , we get  $f(r, 0) = 2\omega^2 r e^{-\omega r}$ , as in fact we should. Moreover,

$$\begin{aligned} \frac{\pi^{1/2} \rho n_0}{2\beta} \int_0^\infty s^{-1/2} \Phi^S A(s) ds &= \frac{\pi \rho n_0 \omega^2}{4\beta} \int_0^\infty (\omega^2 - 2s) \exp\left[s \ln \Phi - \frac{\omega^2}{4s}\right] \frac{ds}{s^2} = \\ &= \frac{\pi \rho n_0 \omega^2}{2\beta} \left( -\omega^2 \frac{d^2}{d\nu^2} - 2 \frac{d^2}{d\nu^2} \right) \int_0^\infty \exp\left[-\frac{\nu}{x^2} + x^2 \ln \Phi\right] \frac{dx}{x^2} \Big|_{\nu=\omega^2} = \\ &= k \exp[-\omega(-\ln \Phi)^{1/2}] \sum_{n=0}^3 d_n (-\ln \Phi)^{1/2 n}, \quad k = \frac{\pi^{1/2} \rho n_0 \omega^2}{4\beta}. \end{aligned} \quad (3.8)$$

Here  $d_n$  are constants which may easily be expressed in terms of the parameters of the problem, if we use the identity

$$\int_0^\infty \exp\left(-\frac{\nu}{x^2} - \mu x^2\right) \frac{dx}{x^2} = \frac{1}{2} \left(\frac{\pi}{\nu}\right)^{1/2} \exp[-2(\nu\mu)^{1/2}].$$

Substituting (3.8) in (3.5), we obtain for the function  $\Psi = -\ln \Phi$

$$\frac{d\Psi}{dt} = a_0 + k \exp(-\omega \sqrt{\Psi}) \sum_{n=0}^3 d_n \Psi^{1/2 n}. \quad (3.9)$$

The solution of this equation is always expressed in quadratures. In many cases it is important to know the behavior of  $\Phi(t)$  at small and large times  $t$ , at different values of the unsaturation, etc. The corresponding formulas are obtained from (3.9) without difficulty. For example, at small  $\Phi(t)$  (complete or almost complete evaporation)

$$\frac{d\Psi}{dt} \approx a_0, \quad \Phi(t) \approx C e^{-a_0 t} \text{ when } \Psi(t) \gg 1.$$

For evaporation close to saturation ( $\Phi(t) \leq 1$ ,  $\Psi(t) \geq 0$ )

$$\frac{d\Psi}{dt} \approx a_0 + k d_0 + (d_1 - \omega d_0) k \sqrt{\Psi}.$$

The problem may be treated in exactly the same way for other types of spectral functions.

So far it has been assumed that the rate of change of the droplet radius can be expressed with the help of (1.1). This expression corresponds to the assumption that the diffusion of vapor from the droplets can be described on the basis of the Maxwell-Langmuir theory, in which the convective diffusion is not considered at all. The use of (1.1) is justified if the turbulent pulsation scales are much greater than the particle dimensions, and the rate of fall of the particles in the medium does not significantly affect the diffusion flux.

**§4. Effect of turbulence on mass transfer kinetics.** In the general case of developed small-scale turbulence expression (1.1) is incorrect. It may be replaced by the relation [6]

$$\frac{dr}{dt} = \frac{\xi D}{\rho \delta} (q(t) - q^*(t)). \quad (4.1)$$

Here  $\xi$  is a coefficient depending on the shape of the particles (for spheres  $\xi = 1$ ), and  $\delta$  is the mean thickness of the diffusion boundary layer.

The turbulence of the medium may be caused either by external (mixing) or internal (motion of particles relative to medium) factors.

If the turbulence is due to external causes, then the characteristic Reynolds number  $R$  is determined by the flow parameters as a whole and in the first approximation does not depend on the dimensions of the droplets, etc. On the other hand, an effective contribution to the convective transfer of vapor molecules from the droplets is made only by pulsations whose scale is less than or close to  $r$  in order of magnitude, i.e., the Peclet number  $P$ , characterizing the relative role of convective motion and molecular diffusion in mass transfer, can be written in the form  $P = vr/D$ , where  $v$  is the characteristic velocity of the turbulent pulsations.

The thickness  $\delta$  of the diffusion layer can be approximated [6] by a power function of the Schmidt number  $S = R/P$ , i.e., instead of (1.1) we get

$$dr/dt = \eta (q(t) - q^*(t)) r^{-\kappa}. \quad (4.2)$$

Here  $\eta$  is a certain parameter depending on  $D$ , and  $\kappa$  is a constant lying in the interval  $(0, 1)$ . The case  $\kappa = 1$  corresponds, obviously, to evaporation in the undisturbed medium, as required by the Maxwell-Langmuir theory, and the case of  $\kappa \approx 0$  to highly developed turbulence, when the vapor transfer is proportional to the surface of the droplet.

As before, we get self-similar spectral functions, which in the given case have the form

$$f_0(r) = 2sr^\kappa \exp\left(-\frac{2s}{1+\kappa} r^{1+\kappa}\right). \quad (4.3)$$

The results of the preceding section, as may easily be seen, are wholly applicable to the case considered if we define the new argument by means of the relation

$$\sigma = 2r^{1+\kappa} (1 + \kappa)^{-1}.$$

Furthermore, we shall consider the influence of free fall of the droplets in the medium, assuming for simplicity that for mass transfer of stationary individual droplets the Maxwellian relation (1.1) is valid. The quantity  $dr/dt$  is satisfactorily described by the

known Fressling formula

$$\frac{dr}{dt} = \frac{D}{\rho r} (q(t) - q^*(t)) \left[1 + 0.27 \left(\frac{\mu}{D}\right)^{1/2} R^{1/2}\right]. \quad (4.4)$$

Taking into consideration that in the given case  $R = wr/\mu$  ( $\mu$  is the viscosity of the medium), and that the rate of free fall  $w$  is proportional to the radius  $r$ , for the self-similar spectral functions we obtain

$$\begin{aligned} f_0(r) &= Cr (r + k^{-1})^{2\kappa/k} e^{-2sr/k}, \\ k &= 0.27 (\mu/D)^{1/2} (R/r^2)^{1/2}. \end{aligned} \quad (4.5)$$

We note the two following obvious ways of generalizing the results of this paper. Firstly, the proposed method can be extended to the case when the transfer kinetics depend not on one, but on many processes of the diffusion type. Secondly, it is possible to extend the method to heterogeneous systems with more than two phases (or components). For example, the method is directly applicable to the analysis of processes in a three-phase water-air system, which is of special interest in connection with the physics of clouds.

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